Slightly altered origins and fuzzy logics

T.I. Wester

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Abstract

In this paper, I analyse Chisholm's paradox as a modal sorites paradox, survey common responses in the literature, and argue that adopting a fuzzy modal logic better preserves the relevant intuitions while avoiding some of the metaphysical commitments present in established responses.

Modal logic – as viewed through the lens of accessibility relations between possible worlds – is an incredibly versatile tool for philosophical reasoning. It allows us to analyse a variety of positions, and many natural-language sentences appear to demand a modal logic for their interpretation. At the same time, calling modal logic a single system is somewhat disingenuous; there are, after all, several modal *logics* each with their own benefits and downsides.

In this paper, I will present a paradox commonly known as Chisholm's paradox: a modal sorites paradox that has been used to argue against certain modal logics (namely logics that accept the S4 axiom), and certain intuitions about modality and identity. After presenting some of the responses to Chisholm's paradox common in the literature, I offer an alternative solution which involves adopting a fuzzy modal logic. I claim that this approach requires fewer metaphysical-, and logical- commitments compared the other responses covered, though a complete argument for adopting fuzzy modal logics is outside the scope of this paper.

1 Background

I will now quickly go over some notions necessary for understanding the arguments in this paper.

A 'possible world' in possible world semantics is – roughly speaking – a way in which things could be. The exact definition varies between thinkers, and certain interpretations are more suited to certain fields, but the basics remain the same. One can start reasoning from the actual world. This is the world from which we evaluate. I can then say "I might have had brown eyes instead of blue ones". What I am doing in saying this is that there is a possible world (which I will call w') wherein I have brown eyes, and that that world is accessible from the actual world (w).

The accessibility relations between worlds are the main elements of variance between the various systems of modal logic. In the world w' I just sketched for example, it seems reasonable that my brown-eyed alter-ego would say: "I could have had blue eyes instead of brown ones". In this way, the world w' must have access to some other world w'' where I have blue eyes. This world w'' could of course be the actual world. In this case the accessibility relationship is symmetrical. Some systems of logic (namely: B and S5) claim that accessibility relations between worlds are always symmetrical.

S4 and S5 are systems of modal logic such that the accessibility relationships between possible worlds are reflexive and transitive. S5 additionally claims that the relationships are symmetrical. S4 and S5 are relatively strong systems, as they impose rules on accessibility relationships that are not present in weaker logics such as S3 and below.

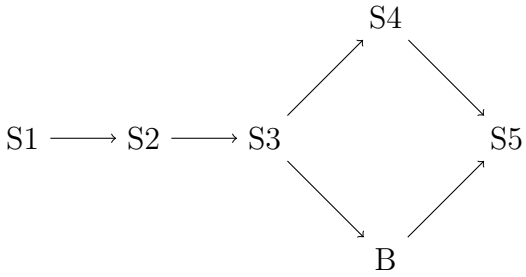

Figure 1: Overview of modal logics

The restrictions introduced by S4 and S5 are very convenient in allowing certain moves in moral reasoning. For example, both S4 and S5 allow one move from $\Box p$ to $\Box\Box p$, while S5 allows moving from $\Diamond p$ to $\Box\Diamond p$. The reasoning relevant to this paper is that both systems allow one to say "If something is possibly possible, then it is possible". This move is due to the transitivity constraint and is thus not possible in B or S3 and below.

While some of the moves allowed by S4 and S5 are useful, they are also problematic. I will specifically show that we have certain intuitions about

possibility that seem to indicate that we do not typically reason with transitive accessibility relations, thus making S4 and S5 unsuitable logics for everyday reasoning.

Aside from the systems of modal logic, and possible world semantics, I will also make use of the notion of rigid designators (Kripke and Munitz 1971). A rigid designator selects the same entity in every possible world. A typical example is a proper name as opposed to a definite description. The name "Iris" refers to me in all possible worlds, while the description "The author of this paper"[1] does not, as there are plenty of possible worlds where some other person than myself had written this exact same paper.

The discussion of rigid designation is a fascinating one, but it is not one that will be explored in this paper. Rather, I will just make use of the notion and assume that proper names are rigid designators.

2 The argument

With those preliminaries out of the way, let me present the main argument. This argument is an adaptation of Chisholm's paradox, with my paraphrasing being inspired by (Williamson 2013) and (Salmon 1993). I specifically phrase the paradox as a modal variant of a sorites paradox because I consider the formulation more clear and because it allows for an easier connection to fuzzy logics.

Consider a table with the rigid designator 'Oaky'. In the actual world, Oaky is made entirely from solid oak. Now consider a possible world wherein Oaky would be made from a different material, say laminated layers of oak. Most people would say that that entity, made from laminated layers of oak, is still Oaky. There is however another table in some possible world that is made entirely out of birch. This table certainly is not Oaky.

However, if we accept that the laminated-oak table in world w' is still Oaky *i.e.*, if we accept that Oaky could be made from a slightly different material, then we encounter a problem. Consider now that Oaky in w' – made from some oak and some glue – could have been made from oak-chipboard. But this Oaky in w'' could have been made from mixed chipboard. That Oaky in w''' could have been made from Birch-chipboard, who could have been made from Birch laminate, who could have been made from solid Birch.

[1] I am glancing over the fact that 'this paper' involves an indexical and would thus involve further variance across possible worlds. Even if were to replace the phrase 'this paper' with the title however, the description still would not select the entity 'Iris' in all possible worlds.

In this way, every step along the way makes only a minor change to Oaky's composition, until we reach a composition which we had determined previously to be impossible. It must therefore be the case that world $w*$, where Oaky is made from solid Birch, is not accessible from w, even though it is accessible from some world accessible from w. The accessibility relationship can therefore not be transitive.

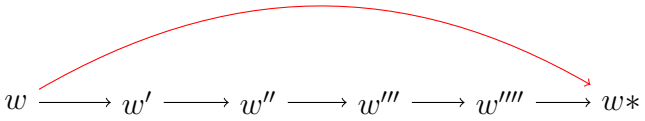

Figure 2: Accessibility relations of possible constitutions of Oaky

The argument can then be formalized as follows:

1. Item x is made from material m_0
2. Item x cannot be made from material m_n
3. Materials m_i and m_{i+1} are sufficiently similar with regard to making up x that it is possible for x made from m_i to be made from m_{i+1}

When formalized in this way, it becomes clear that the argument is a modal sorites paradox. The original formulation of the sorites paradox concerns whether a heap of sand is still a heap when a single grain is taken away. If this premise is accepted (the equivalent of premise 3 in the formulation of the argument above), then one must accept that even a single grain of sand (or indeed grain of sand at all) is still a heap; since if c_i is a heap, then c_{i-1} is a heap as well.

3 Theoretical Consequences

If the validity of the foregoing argument is accepted, it seems that Oaky *can* be made from solid birch by premise 3, but premise 2 rejects this very conclusion. There are broadly four ways of resolving this issue. The first three options are to reject any of the three premises of the argument. These would respectively be the standpoints of Plantinga (section 3.3), Lewis (section 3.2), and Williamson (section 3.1). Alternatively, one can accept each of the premises but work around the paradox by adopting a different logic. One such response is given by Salmon (section 3.4), who calls on rejecting transitive accessibility relations, and I put forward a similar response in section 4 calling on the adoption of a fuzzy logic.

3.1 Rejecting slight material difference

Williamson (2013) takes the approach of rejecting premise 3: that Oaky could be made out of some material m_{i+1} that is ever so slightly different from solid Oak.

Williamson argues that, while Oaky in w_0 and w' are *indiscriminable*, they are not *identical*. The inference that Oaky could be made from material m_{i+1} from the fact that we find it difficult to distinguish Oaky in w_0 from the entity in w' is therefore illegitimate. Discriminability is not a transitive relationship, and this is what our intuitions report about Oaky. But *identity* is transitive, so we need not alter our logic to deal with the paradox.

In the case of Oaky the table, we define Oaky in w_0 as being manufactured in a certain place, at a certain time, and from a certain material. We can fix the identity of the craftsman as well, alongside any other details of Oaky's manufacture. Any table where any of these factors is different may be *similar* to Oaky, it may indeed be *indistinguishable* from him, but "the failure of at least one identity" – the one in w^* – "means that the almost exact similarity of processes of manufacture is insufficient for the identity of their products" (Williamson 2013, p. 130).

Williamson thus fully accepts that we intuitively do not use transitive relationships for determining whether an entity is "the same" to another.

The downside of this approach is that it separates the intuitive notions of possibility and necessity from the logical ones. We can speak about entities across worlds *as if* they were the same, but *metaphysically*, they are not. Williamson's theory furthermore requires that we adopt some mind-independent metaphysical identity, and that possible changes to that object may or may not preserve that identity. Since our intuitions do not necessarily align with metaphysical reality, our intuitive sense of how an object might have differed can fail to track how it really could have been.

3.2 Intransitivity only holds for certain modalities

Lewis (1986) argues that Chisholm's paradox holds – and that transitivity must be rejected – only for certain restricted modalities. The modality that we should really care about according to Lewis is metaphysical modality, which is – according to him – unrestricted.

Take epistemic modality as an example. Epistemic modality is used in statements that make reference to 'something that is known'. For example, in the sentence "it must be raining", the speaker is claiming that they have sufficient knowledge to claim that it is raining. The modal term 'must' then is a modal over a set of worlds compatible with one's epistemic state, with that

one knows[2]. Now, consider that the speaker is in a windowless-, soundproof-room, with no information coming in from the outside. Their epistemic state is such that it contains worlds where it is raining, and worlds where it is not. Both states are compatible with what I know, meaning that "it might be raining".

$$\neg\Box[\![\text{raining}]\!] \wedge \neg\Box\neg[\![\text{raining}]\!]$$

$$\Diamond[\![\text{raining}]\!] \wedge \Diamond\neg[\![\text{raining}]\!]$$

Consider now two worlds w' and w^* compatible with the speaker's epistemic state in w_0 which are respectively rain-worlds and not-rain-worlds. In neither world does the speaker have sufficient knowledge to make a claim in either direction however. As such, w' and w^* can access each other and w_0. The worlds w'' and w^{**} are worlds where the speaker has knowledge to claim "it must be raining" and "it must not be raining" respectively, meaning that w'' has access to w', but no transitive access to w^*. Likewise, w^{**} can access w^*, but not w'.

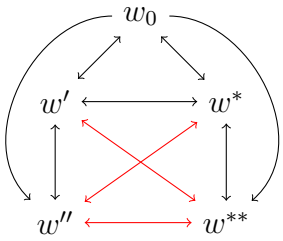

Figure 3: Possible accessibility structure of epistemic modality

In this example, while "it is raining" is an *epistemically necessary* proposition in w'', it is not *metaphysically necessary*. That is, if we do not *restrict* the accessible worlds to those compatible with the speaker's epistemic state, then worlds w^* and w^{**} *are* accessible. The critique then continues that leaving aside certain worlds by restricting our modalities is a "Small comfort" (Lewis 1986, p. 248), since metaphysical modality does not allow us to ignore such worlds.

This critique holds for Oaky the table if we take premise 2 – that Oaky cannot be made from solid Birch – as a restriction on accessibility relations.

[2]Here I am following a fairly standard Kratzer (1991) analysis of 'must' as a necessary modal over an ordered set of epistemically accessible worlds. 'Must p' then is interpreted as saying: The set of best worlds among the ordered set of worlds compatible with my knowledge in the actual world is a subset of the worlds wherein p holds.

As Lewis puts it: "far-out worlds are left aside as inaccessible. Leaving them aside, indeed you could not have been a poached egg" (Lewis 1986, p. 240).

There are two problems with this critique however.

The first problem is that this critique relies on the idea that metaphysical modality is the only one we care about because it is unrestricted. It is a "Small comfort" that Oaky could not be made from Birch according to the restriction imposed by premise 2 because Oaky could *metaphysically* still be made from birch, or pine, or – for that matter – be a poached egg. Salmon (1989) raises the concern that certain modalities are actually *less* restricted than metaphysical modality. Mathematical modality for instance only requires that the laws of mathematics are followed. Thus – while metaphysically *impossible* – it is mathematically possible for "Nathan Salmon [to be] a Visa credit card account with the Bank of America" (Salmon 1989, p. 13). Our choice of metaphysical modality over all others then, is at best arbitrary, and at worst needlessly restrictive.

The second problem is easier to see if we look at the original formulation of the sorites paradox – where grains of sand are repeatedly removed from a heap until no heap is left – one could claim that our intuition that "zero grains of sand do not make a heap" is an arbitrary restriction on metaphysical modality. That *metaphysically* there is no such criterion for heaps. Doing so brings the equal and opposite problem of rejecting premise 3. Williamson's response meant committing to a realism of identity, while Lewis' response seems to require a commitment that identities are entirely unreal. This once-again distances the way in which we intuitively speak about how an individual might have been different, and how we logically speak about such differences.

3.3 Rejecting material origins

Plantinga (1978) can be seen rejecting premise 1 in a certain sense. While he does not claim that Oaky is not made from solid Oak, he rejects the idea that this property could be in any way what defines Oaky. Plantinga claims that there is some essential property of an object, and that this property need not be apparent to us. Indeed, many of the properties that *are* apparent do not seem to be essential to the entities to which we attribute them, as we can imagine the same entity with different properties.

For Plantinga, the essential property of an entity x is one such that x has that property in all worlds where x exists, and only x ever has this property across all possible worlds. In other words, the property in question is a rigid designator. The fact that we cannot identify this property or use it to pick

out an individual in a world is unimportant for Plantinga who claims that "[t]o suppose that Cantor was a precocious baby at t it is not necessary that I be able to pick his picture out of a gallery of babies-at-t" (Plantinga 1978, p. 95).

If we return to the example of Oaky then, for Plantinga, if the essence of Oaky is maintained when he is made from solid Birch, then he is Oaky in world w^*. Even if we cannot recognize him as Oaky. Indeed, merely phrasing a statement such as "Oaky could have been made from solid Birch" is sufficient to select a world where Oaky (however unrecognizable) is made from solid birch, thereby proving that our original identification of Oaky as somewhat tied to his solid-oak construction was wrong. Compare "Oaky if he was made from birch" to a statement like "Okay if he was not Oaky". This is clearly a much stranger statement, and we could rightfully say that that world does not exist, or at best, that it is a world where Oaky does not exist.

The problem with this approach is the price it requires us to pay with regard to intuitive notions of what an entity is. It may seem insignificant whether Oaky could be made of Birch, but what if we consider the world where Socrates is not a Philosopher or not an Athenian, or not a person who asked a lot of questions. What if we consider the world wherein he is none of these. I am inclined to agree with Searle (1958) that entities necessarily have the logical sum of properties commonly attributed to them (Searle 1958, p. 172). So that Socrates could have varied on *any* of the properties listed above but not on *all* of them. As we have defined Oaky only as an oaken table, it seems essential to him that he retains at least some oakiness and some tableness.

For Plantinga however, neither commonly attributed properties nor their logical sum are essential to a given entity. "These properties are ones that I may use to get you to see about whom it is I am talking", but "that these properties are essential to him does not follow." (Plantinga 1970, pp. 467-8). This once-again separates how we intuitively speak about possibility and necessity from how our logical systems account for it.

3.4 Rejecting transitivity

Salmon's (1989) answer to the paradox is to reject transitivity as a requirement on accessibility relations, and with it reject S4 and S5 as viable modal logics. This allows Salmon to fully accept slight material difference, without thereby needing to accept that Oaky could be made from solid Birch.

Take our position in w_0. From here we can access w', w'', and maybe w'''. As such, we can accept that Oaky could be made from solid oak, oak

laminate, oak chipboard, and maybe mixed chipboard. While from w''' where Oaky is made from mixed chipboard, we can also access w'''' where Oaky is made from birch chipboard. Since the relationship is not transitive, w_0 need not access w'''', a conclusion compatible with our intuitions.

This approach makes the question of "what materials could Oaky have been made of" is relative to the world wherein the question is asked. At the same time, the actual world is not arbitrarily privileged over others.

By getting rid of transitivity however, Salmon allows for two primary consequences: weakening our logic, and allowing for impossible worlds.

With regard to weakening logic, B and weaker logics allow for fewer valid inferences to be made. Returning to the epistemic domain, we see for instance, that positive introspection is lost, allowing for someone to know that p without knowing that they know that p. Rejecting positive introspection may be desirable however, as it is far from universally accepted. Gopnik (1993) for instance points out that introspection arises developmentally at the same time as theory of mind, suggesting that access to our own beliefs is not direct and transparent, but depends on interpretive capacities closely related to the capacities we use to understand other minds (Gopnik 1993).

With regard to impossible worlds, rejecting transitivity allows 'possibly possible but impossible worlds' into our reasoning. This is seen by some (such as Plantinga (1970; 1978)) as a very unintuitive conclusion, but is seen as intuitive by others (such as Chandler (1976)). Impossible worlds are not necessarily problematic. Berto (2017) for instance introduces impossible worlds into logic to explain how *imagination* can be used in logical reasoning while also being hyper intentional in nature (Berto 2017).

Thus, while the previous responses separated intuitive speech from logical rigour, Salmon's approach instead weakens logic until it matches intuitive speech. I will now present an approach that does neither.

4 Adopting a fuzzy logic

My proposed solution for dealing with Chisholm's paradox is to adopt a Gödel-Dummett-style fuzzy logic (henceforth 'fuzzy logic'). In such a system, truth values, rather than being strictly binary, exist on the real unit interval $[0, 1]$. This means that statements can be true or false *in degrees* allowing for certain statements to be *more true* than others.

For a regular sorites problem, starting from a heap of 10.000 grains (which – for the sake of this argument – I will take to be a heap with truth-value 1), each grain that is removed reduces the truth value of the statement "this collection of grains is a heap", so that we eventually reach a point where the

collection is not a heap at all. The reason this works is that the classification of the collection as a heap is not preserved for every grain-reduction.

For extending fuzzy logics to the modal domain, I will be following Godo (1999) in adopting the notion of 'truthlikeness' where "the degree of truthlikeness of a sentence ϕ depends on the similarities between the states of affairs allowed by ϕ and the true state of the world" (Godo and Rodríguez 1999, p. 2). In other words, the truth value of the statement "This is Oaky" at any world w is dependent on the similarity between the referent of 'this' in w and Oaky in the world of evaluation. This is *not* a claim about the metaphysical identity of Oaky however, so the world of evaluation is not privileged over any other. Indeed, for something to be truth-like, it need not be "true or even probable" (Godo and Rodríguez 1999, p. 2)

Returning to the paradox then, the statement that "this table is Oaky" is true with a truth value of 1 in w_0 where Oaky is made from solid oak, and 0 in w^* where the table is made from solid Birch. These values are taken from premises 1 and 2 respectively. Oak laminate, oak chipboard, mixed chipboard and birch chipboard tables respectively receive truth-values like 0.8, 0.6. 0.4, and 0.2[3]. In other words, the statement "This table made from oak chipboard is Oaky" is 60% truth-like when evaluated from w_0.

In this way, fuzzy logics allow for modeling our intuitions regarding the materials that may constitute Oaky without rejecting transitivity nor requiring strong commitments about the metaphysical nature of what Oaky is.

The approach is in some ways similar to Forbes' (1985), who claims that certain entities – namely artefacts and composite organisms like slime moulds – have 'fuzzy essences (Forbes 1985, p. 185), meaning that their *metaphysical* identities are tolerant to minor changes. The tolerance principles which Forbes introduces then are "almost wholly true" (Forbes 1985, p. 173), meaning true to some high degree on the $[0, 1]$ interval.

The difference between my approach and Forbes' is that his claims apply to metaphysical identity, while my approach does not require this commitment. This allows metaphysical identity to be crisp (*i.e.*, non-fuzzy). The exploration of whether metaphysical identity is fuzzy or crisp is outside the scope of this paper however.

That being said, the fact that my approach places the lines between entities as an arbitrary convention of *language*, rather than an arbitrary definition in *metaphysics* is – in my mind – a positive outcome. Whether or not we call something a heap or not seems to depend on a number of

[3]These values are merely illustrative, though a principled method of determining fuzzy truth values can of course be developed, see for instance (Godo, Esteva, et al. 1991)

contextual and individual factors subject to constant change. To claim that there is a metaphysical line drawn between heaps and non-heaps, seems to me a more costly commitment than saying that what we call a heap is an arbitrary convention while heaps metaphysically exist only as greater-or-lesser extents. This approach also allows for a more uniform treatment of terms such as 'heap' and 'Oaky' and typically gradable terms such as 'tall' and 'heavy'. The possibility of covering these terms under a single theory – while not explored in this paper – is another benefit in my mind.

In conclusion, adopting a fuzzy logic in response to Chisholm's paradox allows one to maintain transitive accessibility relations contrary to Salmon's approach, while avoiding Williamson and Plantinga's metaphysical rigidity and Lewis' claims about unrestricted metaphysical modality. Fuzzy modal logics do come with their own challenges however, and important technical questions remain (see for instance (Dastgheib et al. 2016; Fitting 1991)). This paper does not answer or address these questions, but rather aims to show adopting a modal logic in response to Chisholm's paradox is a strong contender compared to common approaches in the literature due to allowing for a more intuitive connection between logic and ordinary speech, and requiring fewer metaphysical commitments.

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